Adaptively-Secure Secret Sharing

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(Joint work with Zahra Jafargholi, Karen Klein, Ilan Komargodski, Krzysztof Pietrzak and Daniel Wichs)

IST AUSTRIA

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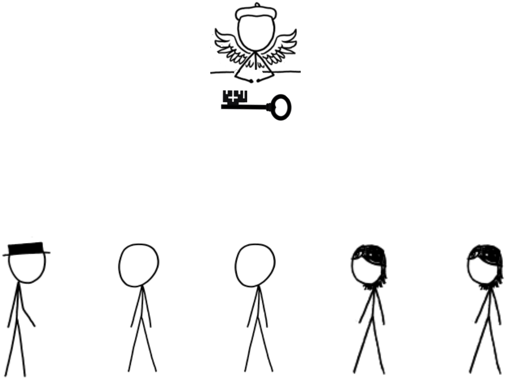
Overview

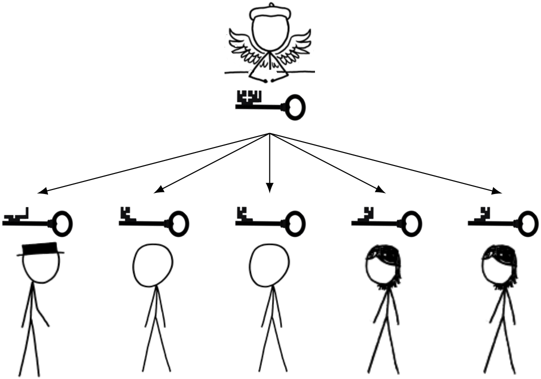
Secret Sharing



xkcd.com

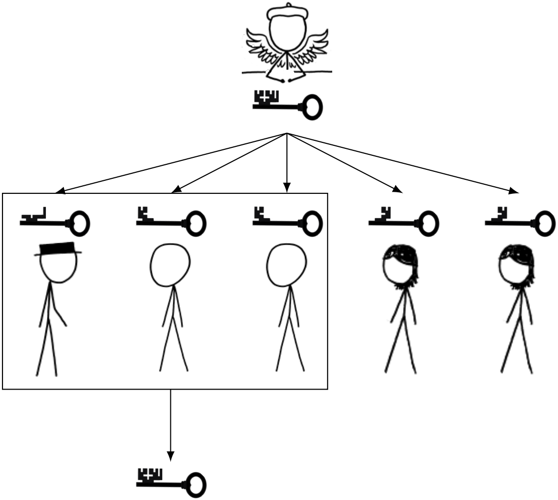
Secret Sharing

1.) Share



xkcd.com

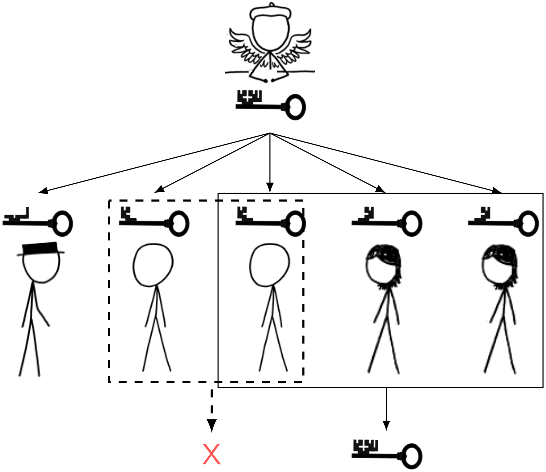
1.) Share 2.) Reconstruct



xkcd.com

Secret Sharing

1.) Share 2.) Reconstruct 3.) Access structure



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Definitons

- $\mathsf{Share}(S) \to \Pi_1, \ldots, \Pi_n / \mathsf{Reconstruct}(\Pi_{\mathcal{X}}) \to S$ for $\mathcal{X} \subseteq [n]$

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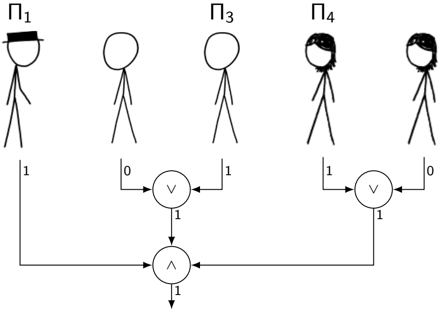

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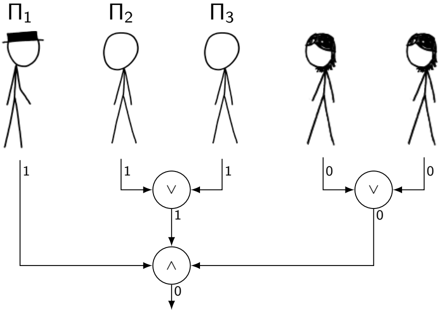

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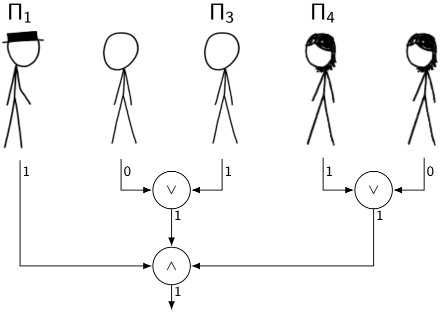

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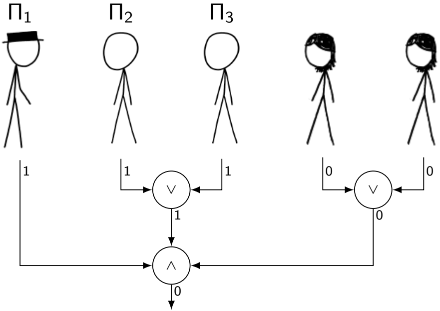
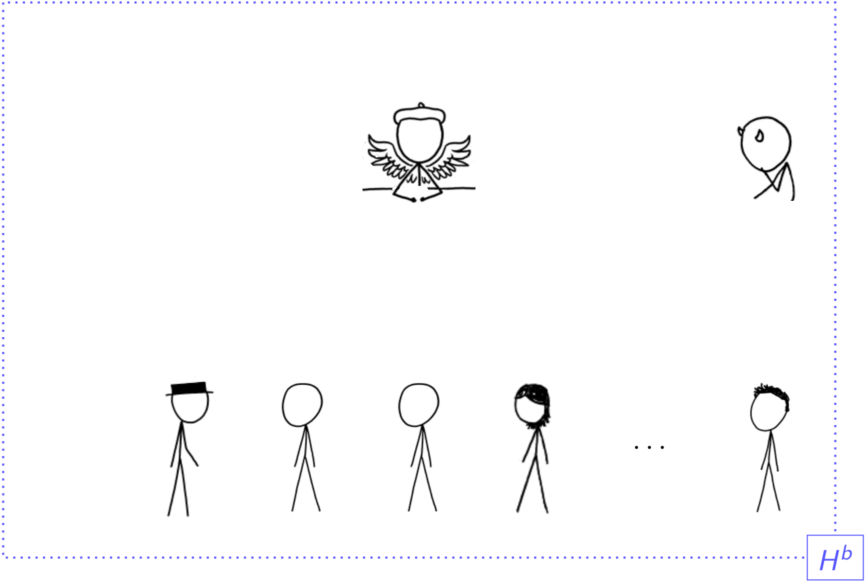


- Completeness: qualified $\mathcal{X}$ can reconstruct
- Security: unqualified $\mathcal{X}$ learns *nothing* about S

Security: Selective Game



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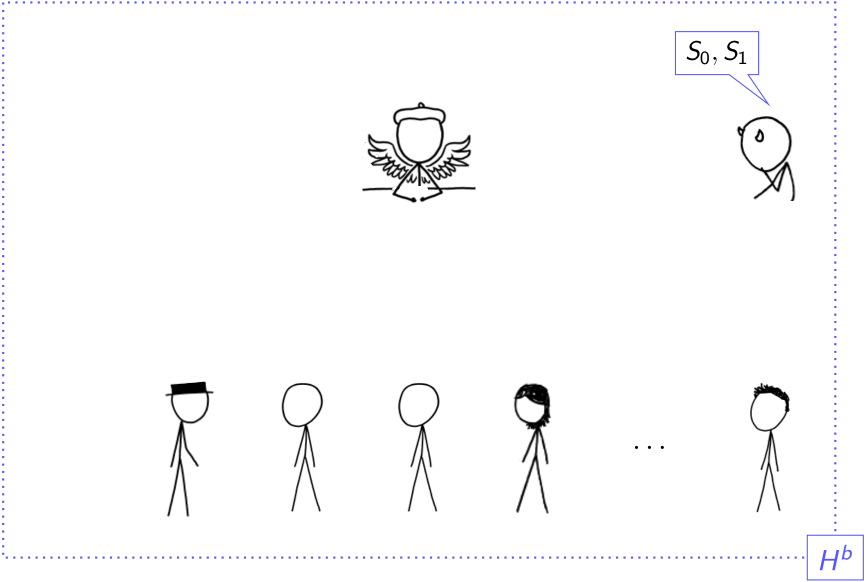

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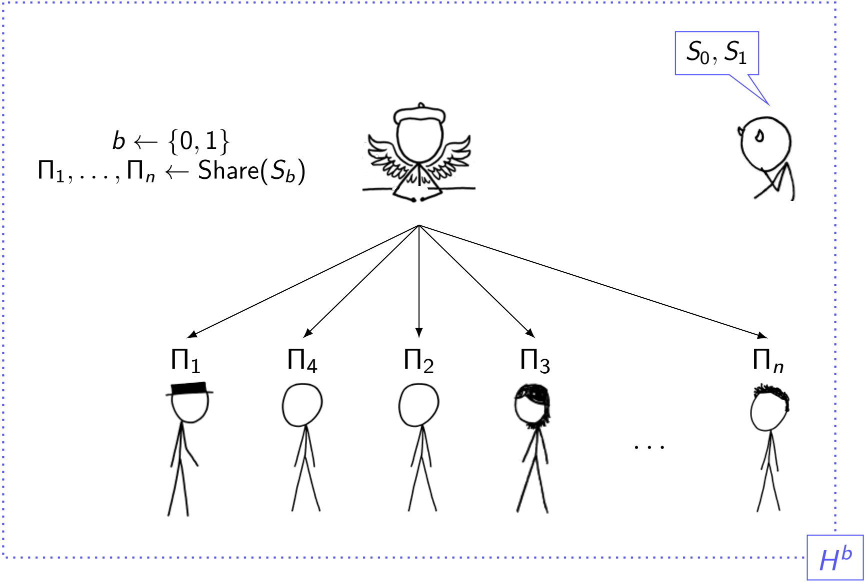

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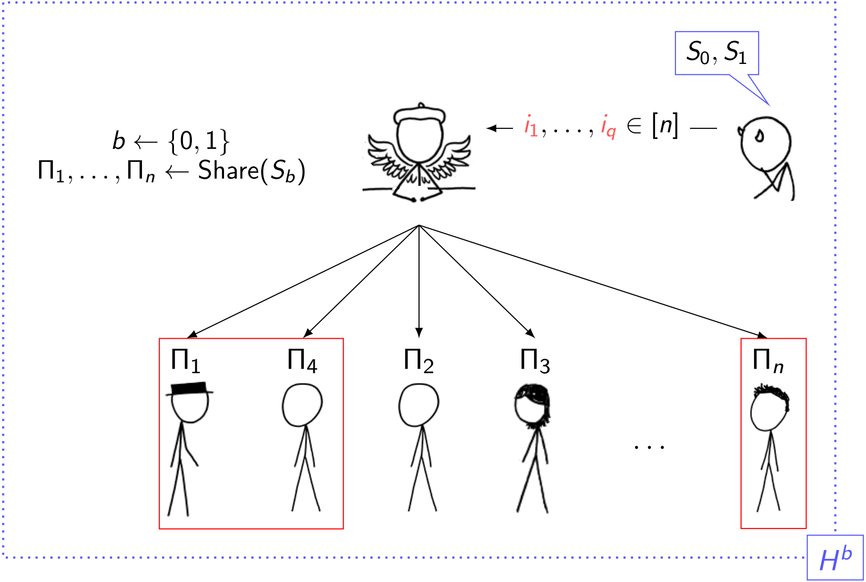

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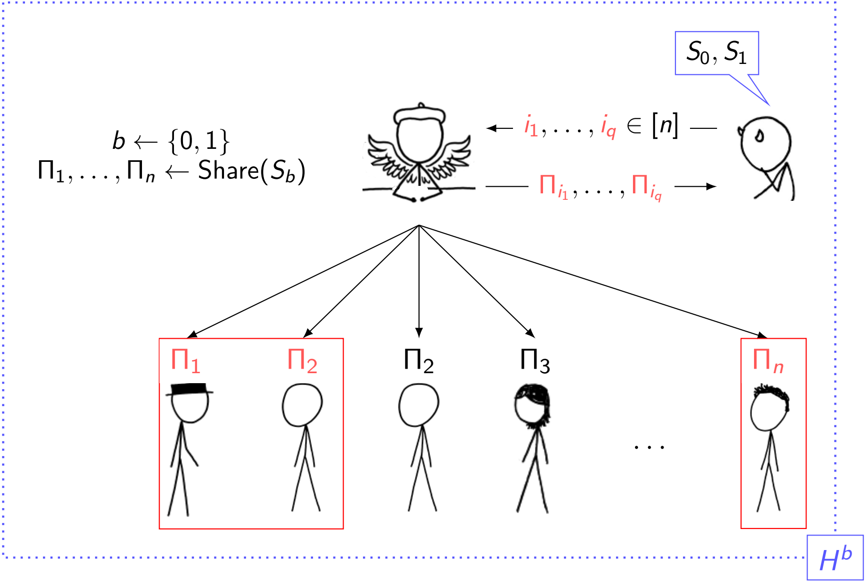

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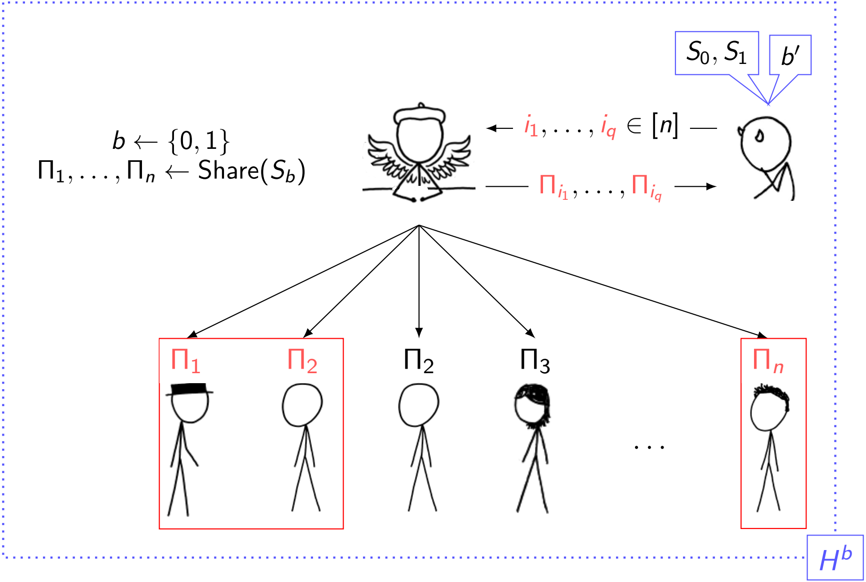

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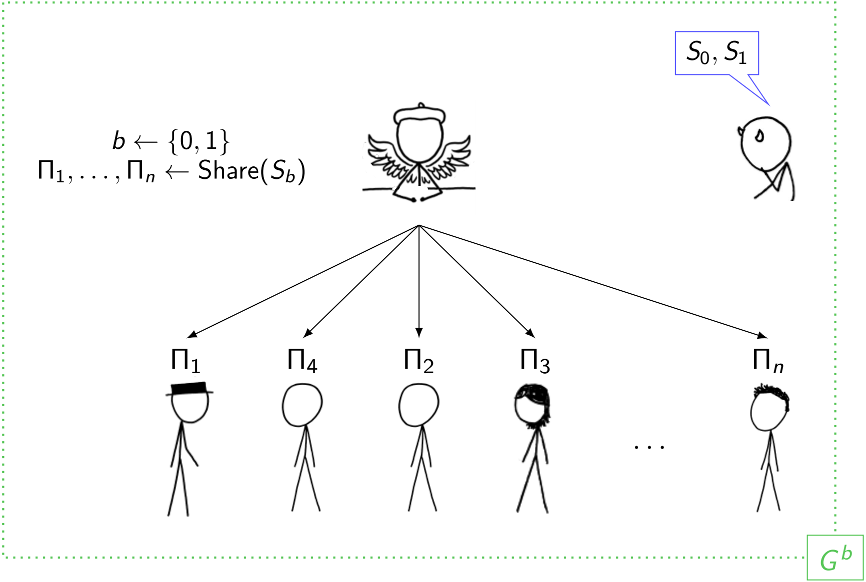

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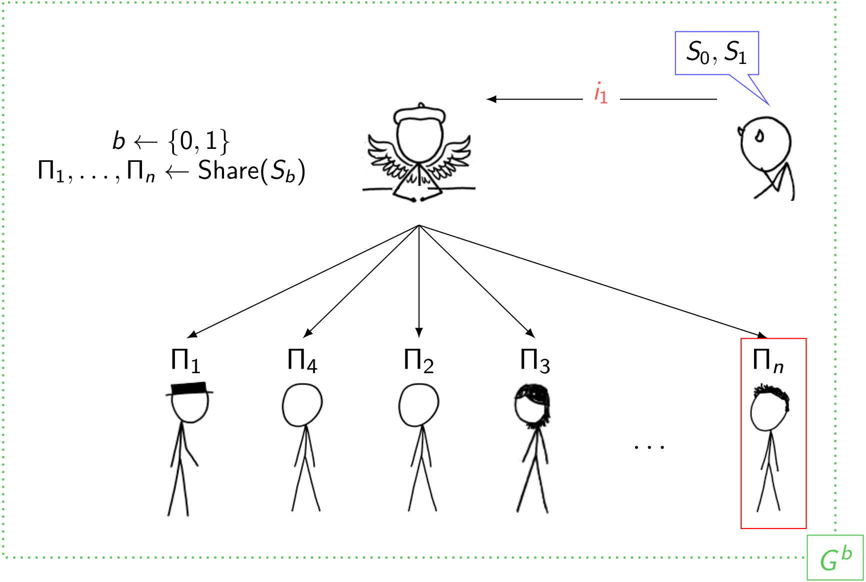

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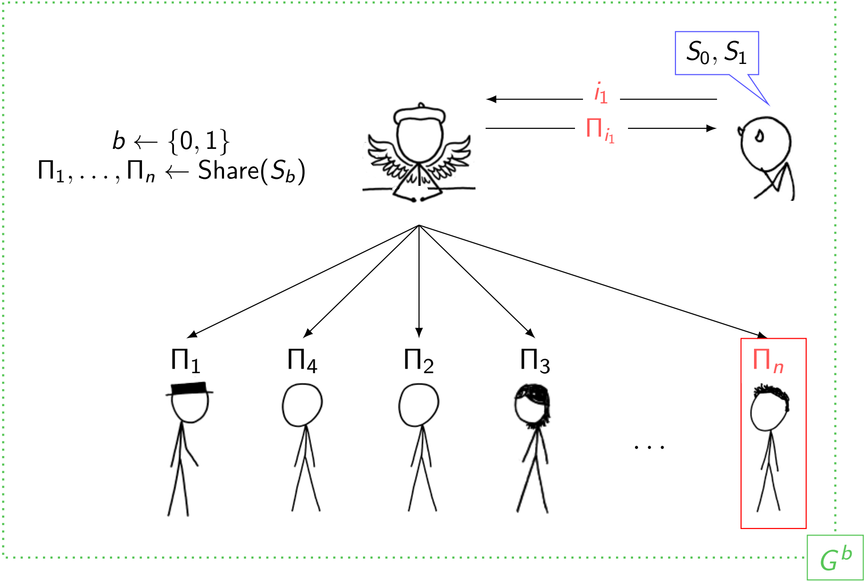

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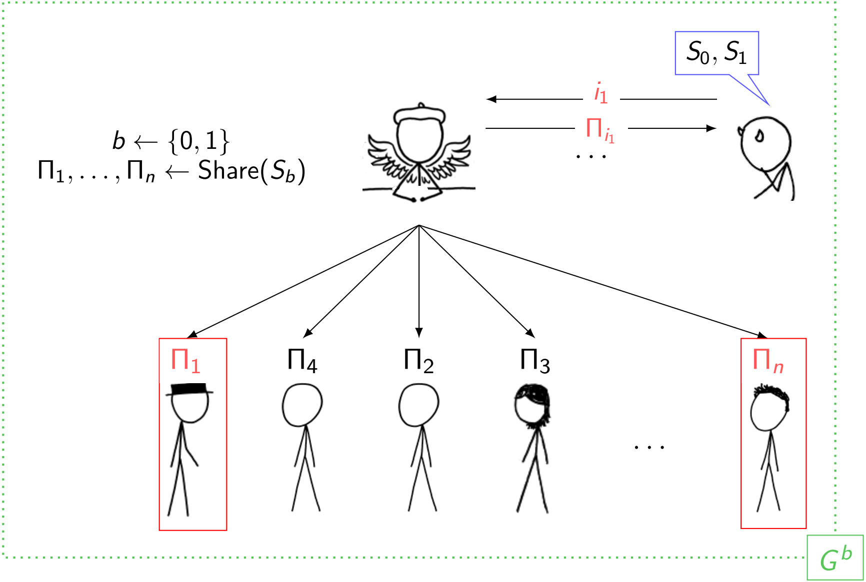

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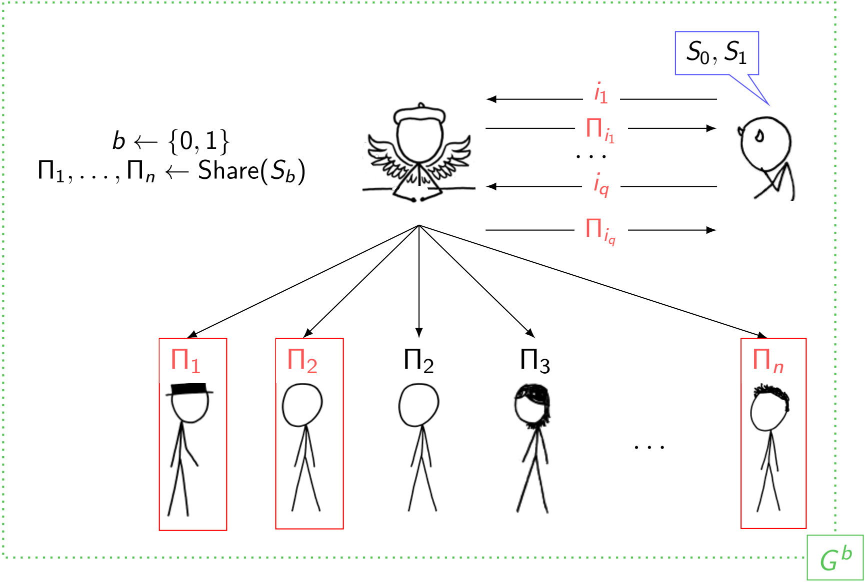

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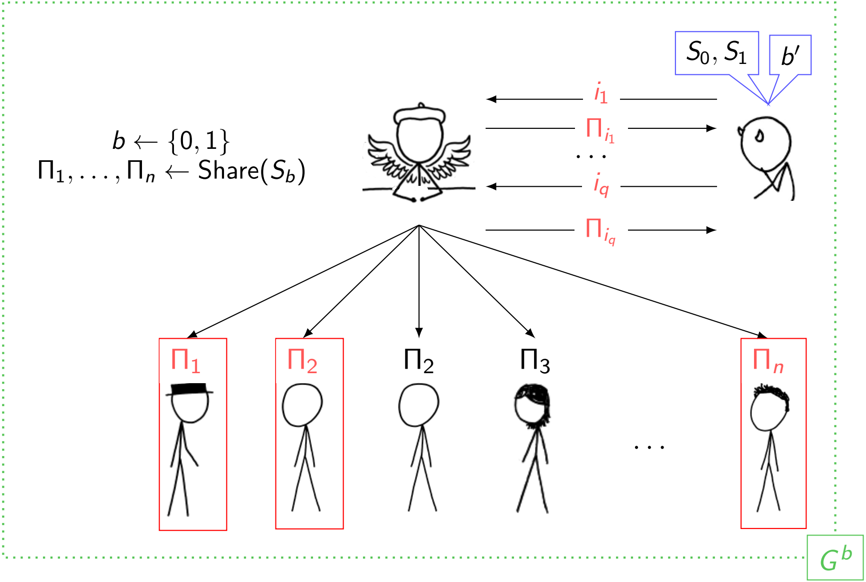

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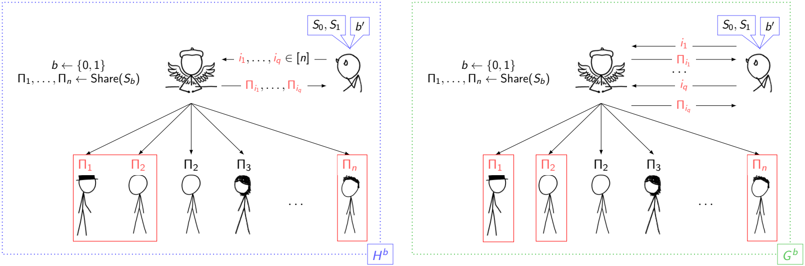

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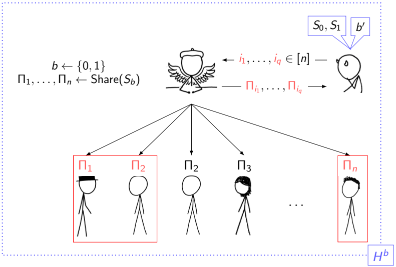
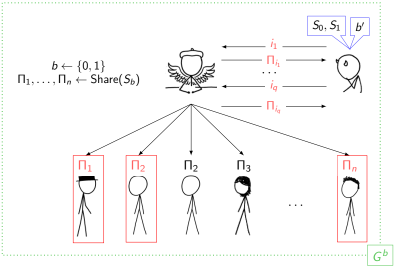
Security...





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- Adversary: computational or unbounded
  - Computationally-secure: ϵ is negligible for all adversaries
  - Negligible function: grows slower than any inverse polynomial
  - Equivalently: G^0 and G^1/H^0 and H^1 are indistinguishable ($\leftrightarrow$)

Selective to Adaptive: Random Guessing

- Lemma 1: ϵ-selective security $\implies$ $\epsilon \cdot 2^n$-adaptive security:
  - Guess the participants that the *adaptive* adversary corrupts
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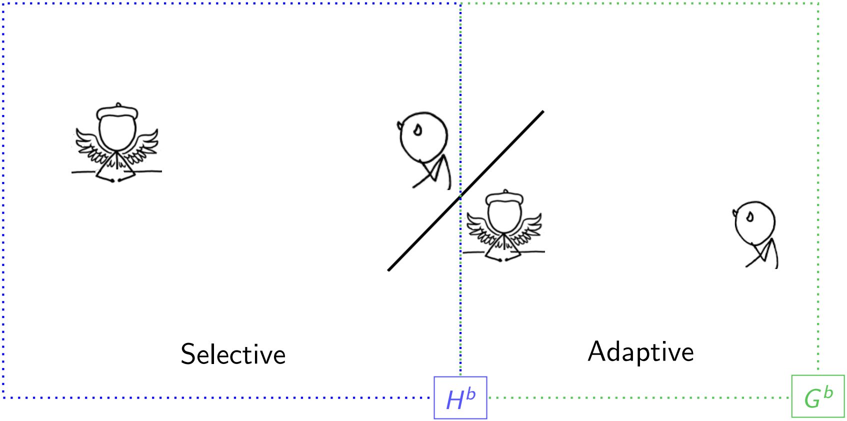

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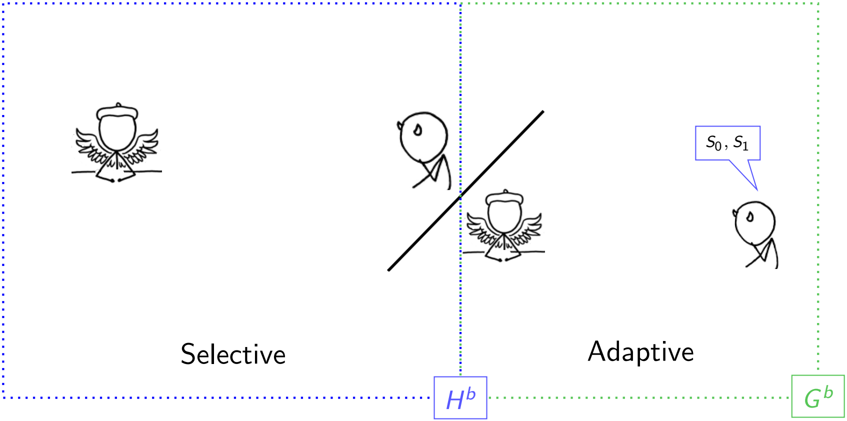

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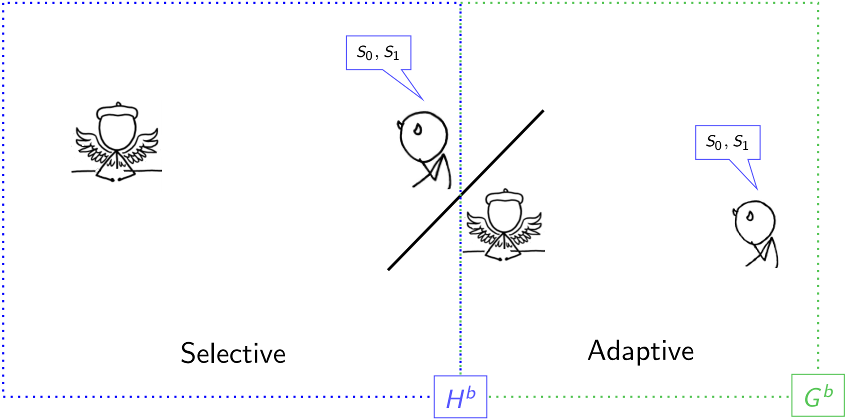

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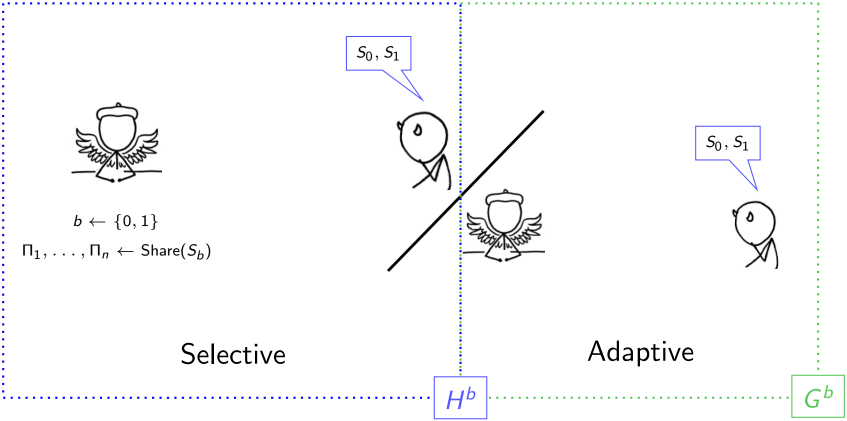

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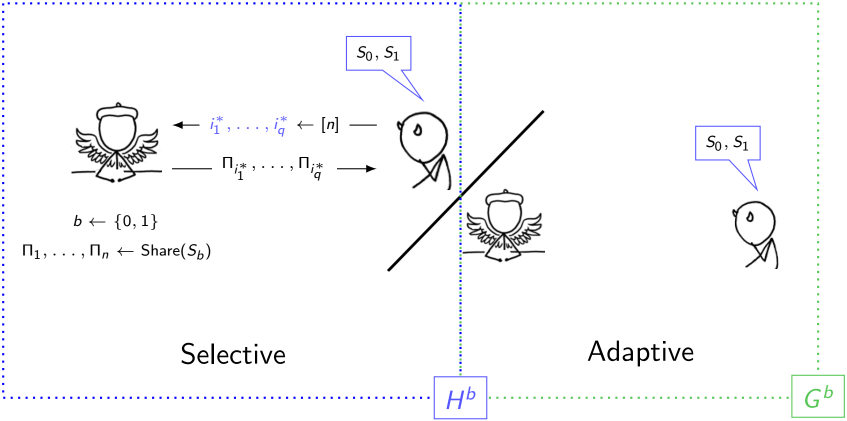

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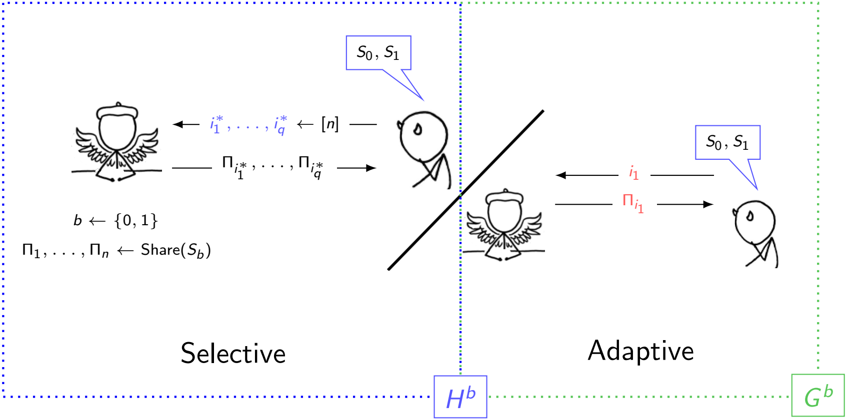

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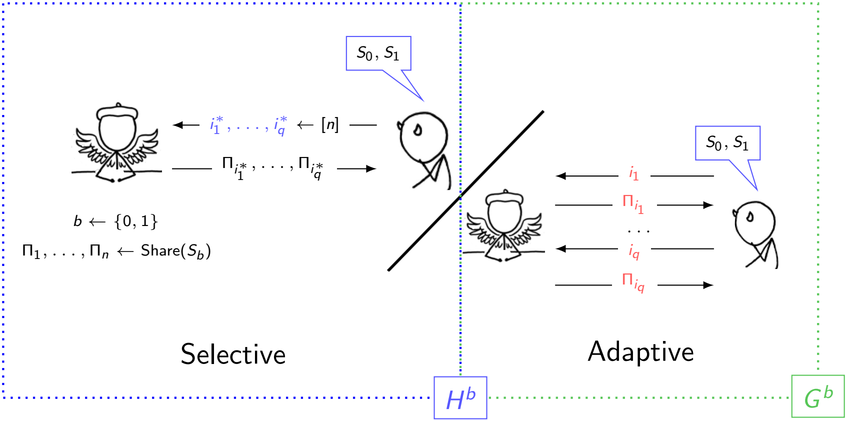

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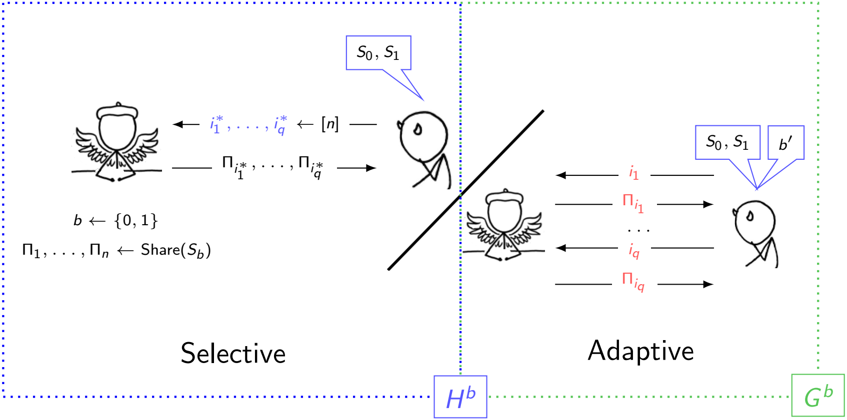

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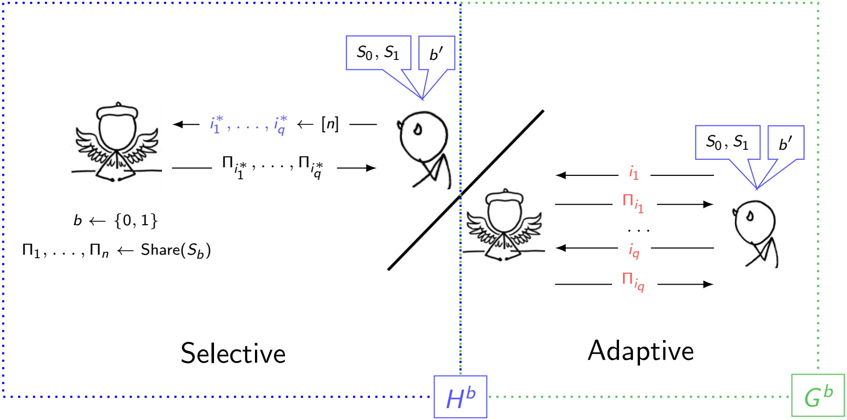

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  - Adaptive security *harder* to achieve:
    - Only known through random guessing

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- Theorem 1: If the encryption is ϵ-secure, then for any access structure described by a Boolean circuit of size s, depth d and fan-in/fan-out δ, Yao's scheme is $\approx \epsilon \cdot (2\delta)^d \cdot s^{\delta \cdot d}$ adaptively-secure

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- Corollary 1: For *log-depth* circuits of *constant* fan-in/fan-out, quasi-polynomially-secure symmetric encryption implies adaptively-secure secret sharing

Yao's Secret Sharing

Yao's Scheme

- Symmetric encryption scheme (E, D)
  - Encrypt $\mathsf{E} : \mathcal{K} \times \mathcal{M} \to \mathcal{C}$ and decrypt $\mathsf{D} : \mathcal{K} \times \mathcal{C} \to \mathcal{M}$

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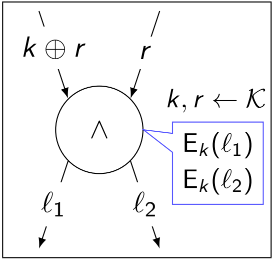

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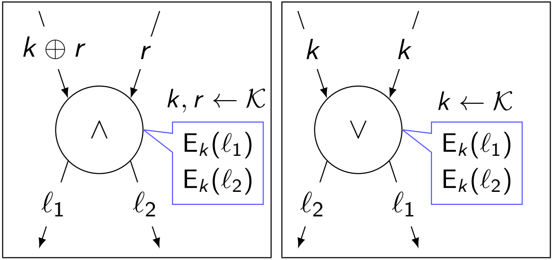

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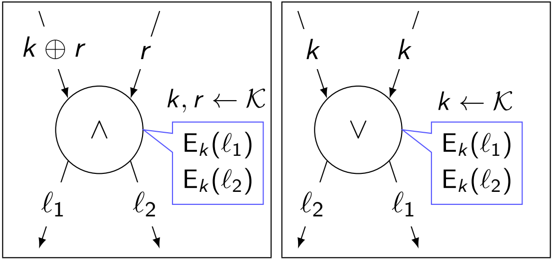
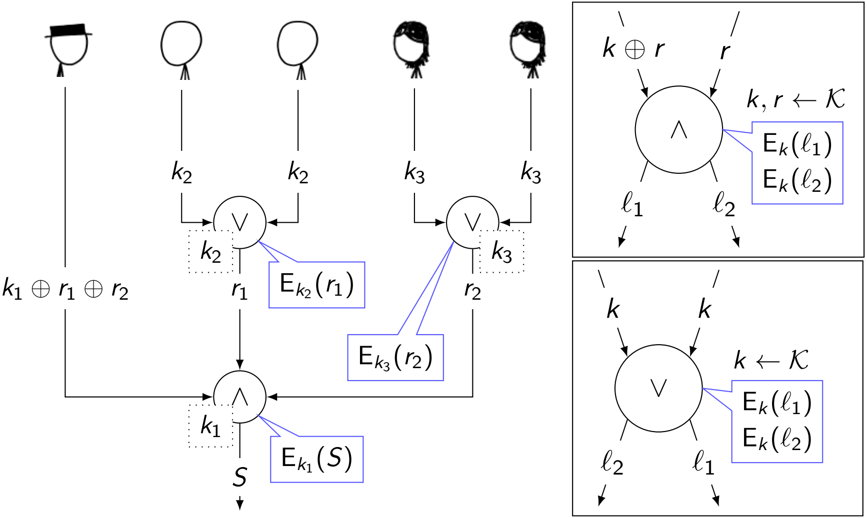


- Reconstruct does the reverse of Share

Yao's Scheme...



k_2
k_2
k_3
k_3
k_2
k_3
E_{k_2}(r_1)
k_1 ⊕ r_1 ⊕ r_2
r_1
r_2
E_{k_3}(r_2)
k_1
E_{k_1}(S)
S
k ⊕ r
r
k, r ← 𝒦
E_k(ℓ_1)
E_k(ℓ_2)
ℓ_1
ℓ_2
k
k
k ← 𝒦
E_k(ℓ_1)
E_k(ℓ_2)
ℓ_2
ℓ_1

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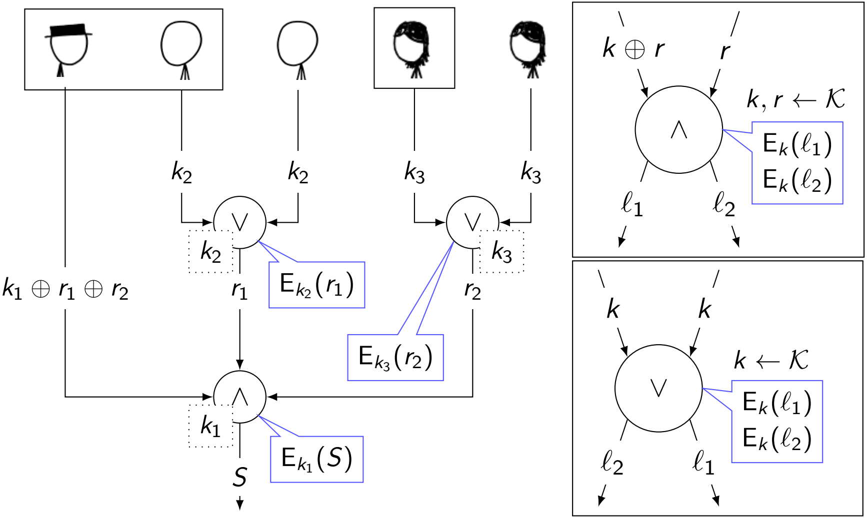
k_2
k_2
k_3
k_3
$\vee$
$\vee$
k_2
k_3
$k_1 \oplus r_1 \oplus r_2$
r_1
r_2
$E_{k_2}(r_1)$
$E_{k_3}(r_2)$
$\wedge$
k_1
$E_{k_1}(S)$
S
$k \oplus r$
r
$k, r \leftarrow \mathcal{K}$
$\wedge$
$E_k(\ell_1)$
$E_k(\ell_2)$
ℓ_1
ℓ_2
k
k
$k \leftarrow \mathcal{K}$
$\vee$
$E_k(\ell_1)$
$E_k(\ell_2)$
ℓ_2
ℓ_1

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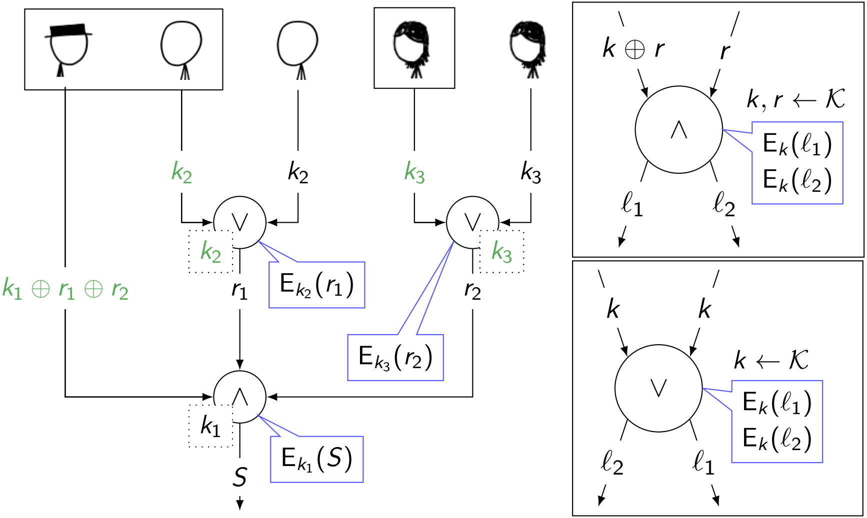
$k_1 \oplus r_1 \oplus r_2$
k_2
k_2
k_2
k_3
k_3
k_3
$E_{k_2}(r_1)$
r_1
r_2
$E_{k_3}(r_2)$
k_1
$E_{k_1}(S)$
S
$k \oplus r$
r
$k, r \leftarrow \mathcal{K}$
$E_k(\ell_1)$
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ℓ_1
ℓ_2
k
k
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ℓ_2
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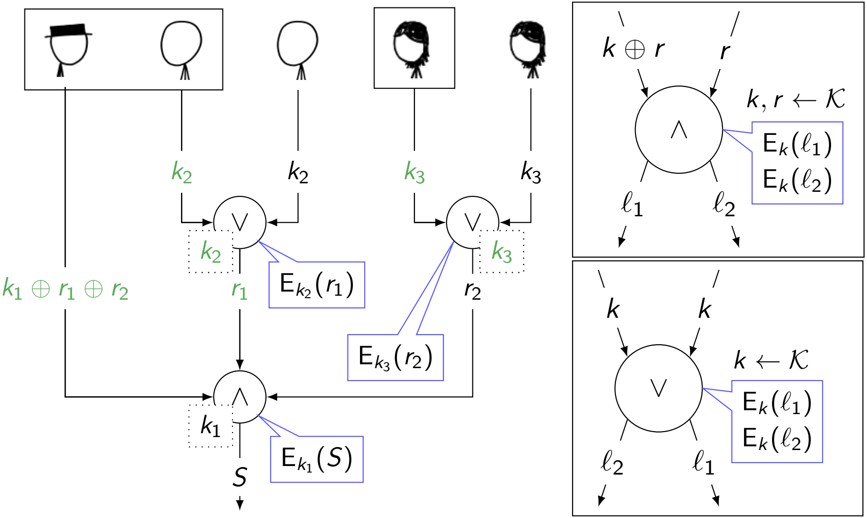

k_2
k_2
k_3
k_3
$\vee$
$\vee$
k_2
k_3
$k_1 \oplus r_1 \oplus r_2$
r_1
r_2
$E_{k_2}(r_1)$
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k_1
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S
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ℓ_1
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k
k
$k \leftarrow \mathcal{K}$
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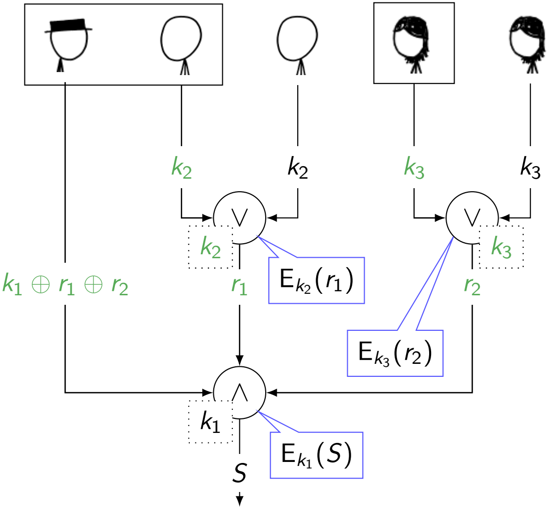
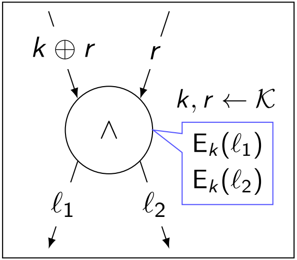


k_2
k_2
k_3
k_3
$\vee$
$\vee$
k_2
k_3
$k_1 \oplus r_1 \oplus r_2$
r_1
r_2
$E_{k_2}(r_1)$
$E_{k_3}(r_2)$
$\wedge$
k_1
$E_{k_1}(S)$
S



$k \oplus r$
r
$k, r \leftarrow \mathcal{K}$
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$E_k(\ell_1)$
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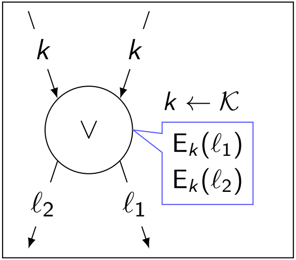
ℓ_1
ℓ_2



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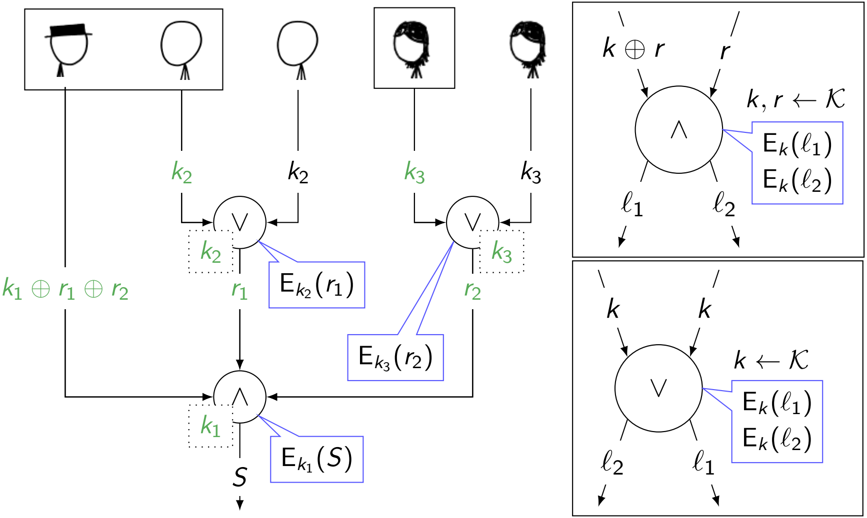
Yao's Scheme...



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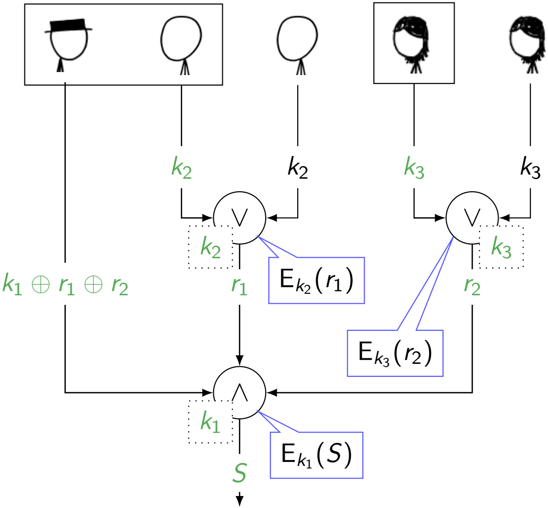

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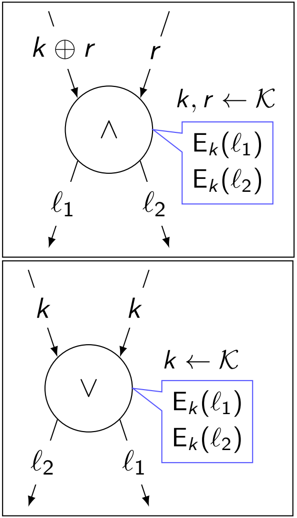

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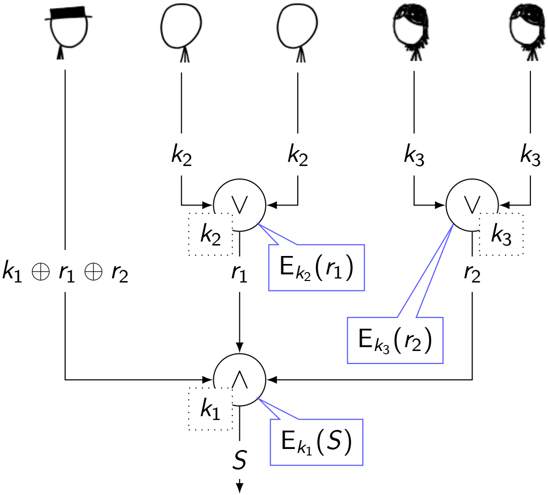

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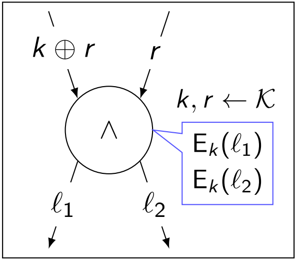

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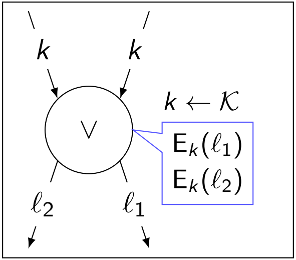

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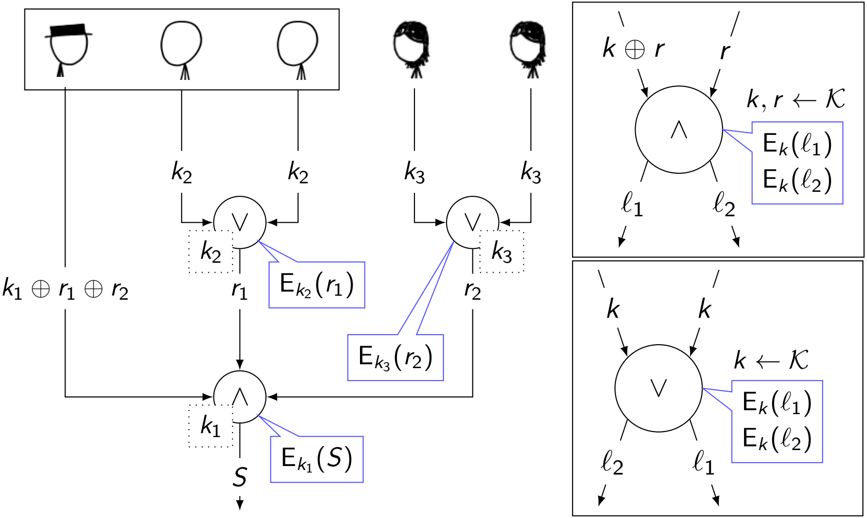
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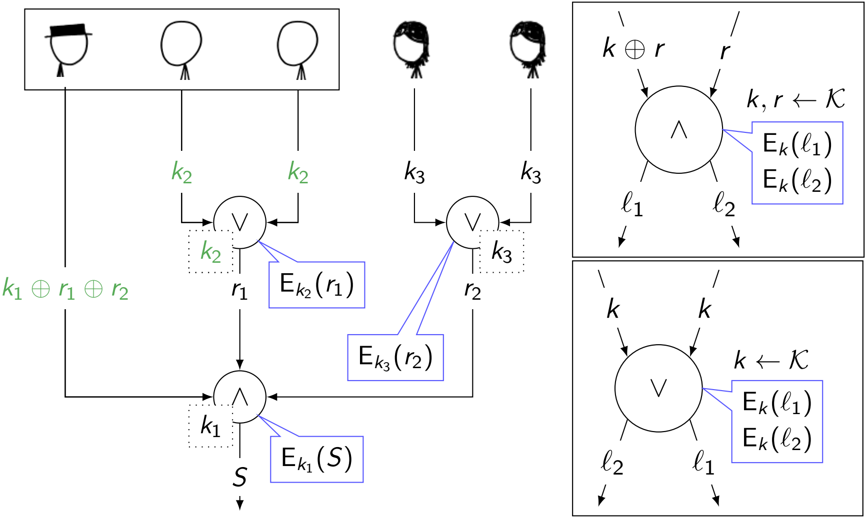

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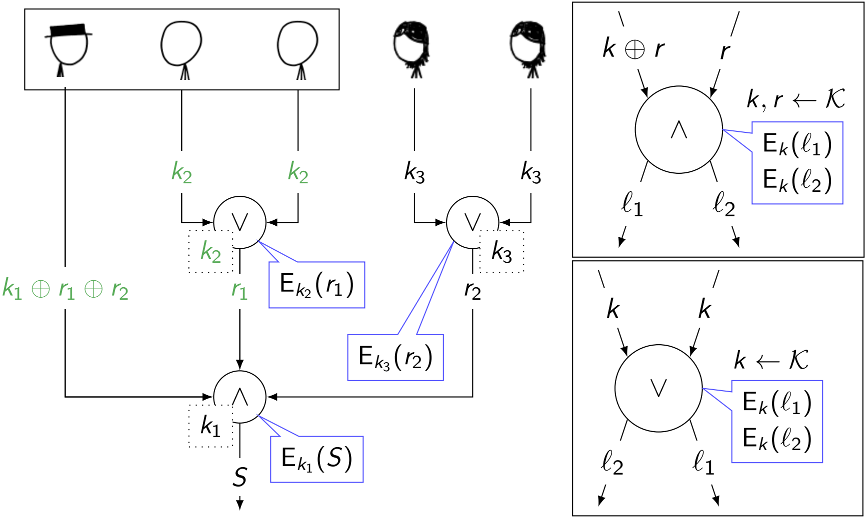

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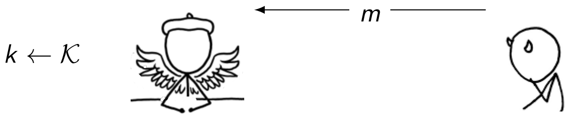

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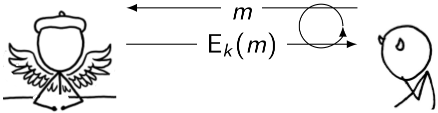

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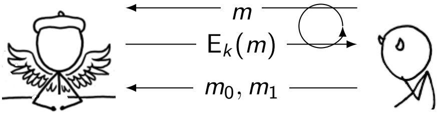

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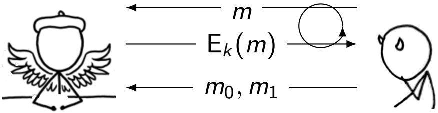

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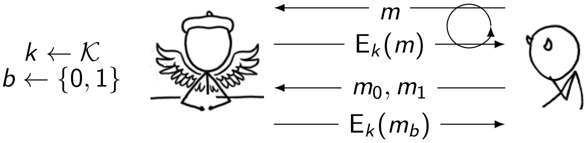

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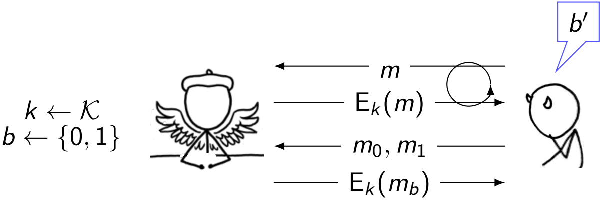

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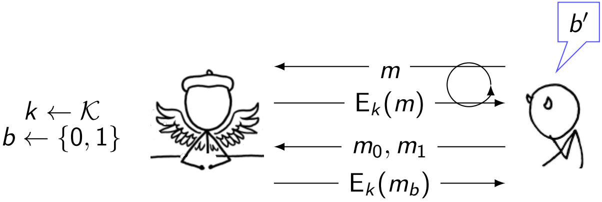

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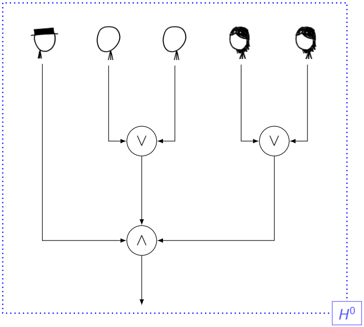
- Theorem 2 [VNS+]: If the encryption is ϵ-secure then for any access structure described by a Boolean circuit of size s the scheme is $\approx \epsilon \cdot s$-*selectively*-secure

Yao's Scheme: Selective Security...



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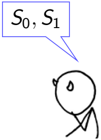

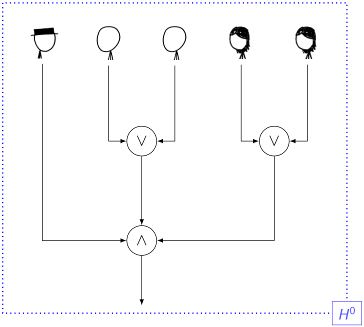

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$b \leftarrow \{0,1\}$
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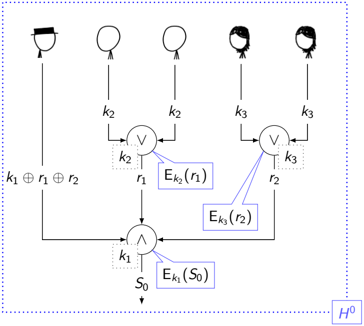

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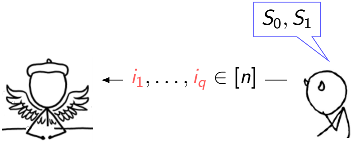

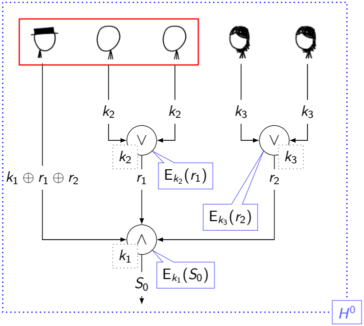

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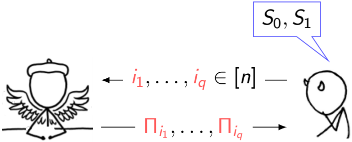

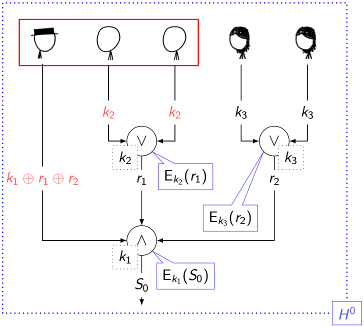

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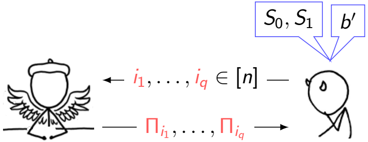

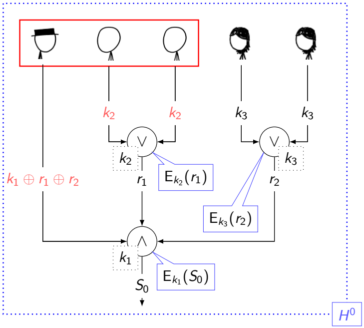

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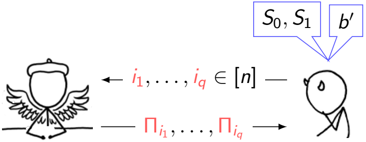

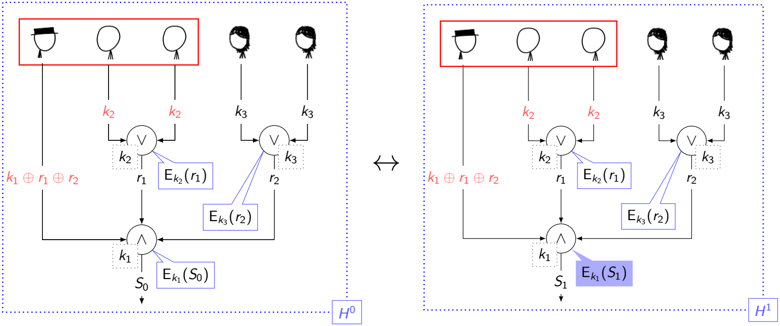

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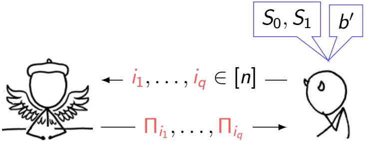
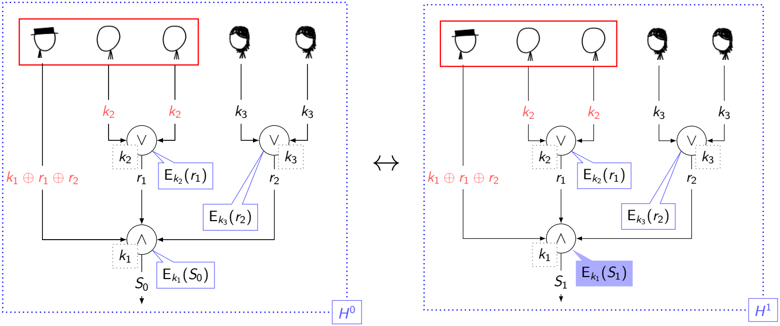
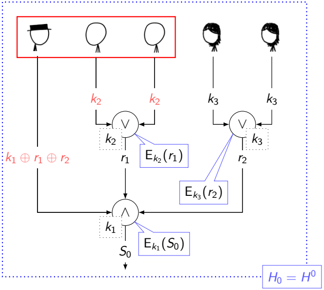
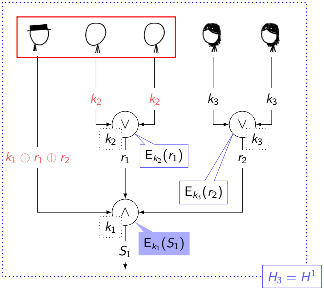




- Aim: Show that secure encryption $\implies$ $H^0 \leftrightarrow H^1$
  - Contapositive: $H^0 \not\leftrightarrow H^1$ $\implies$ encryption not secure

Yao's Scheme: Selective Security...





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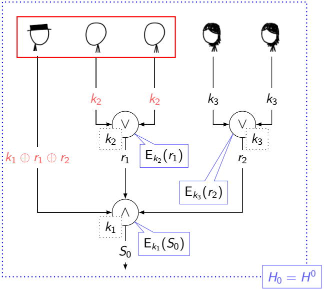

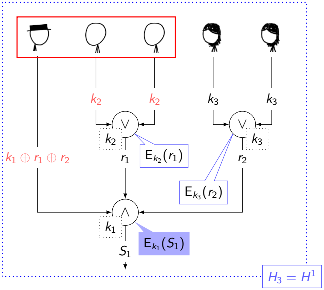

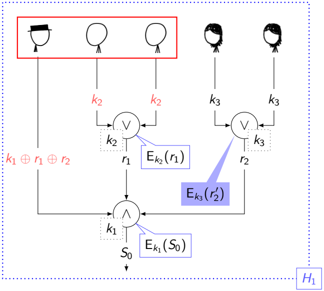

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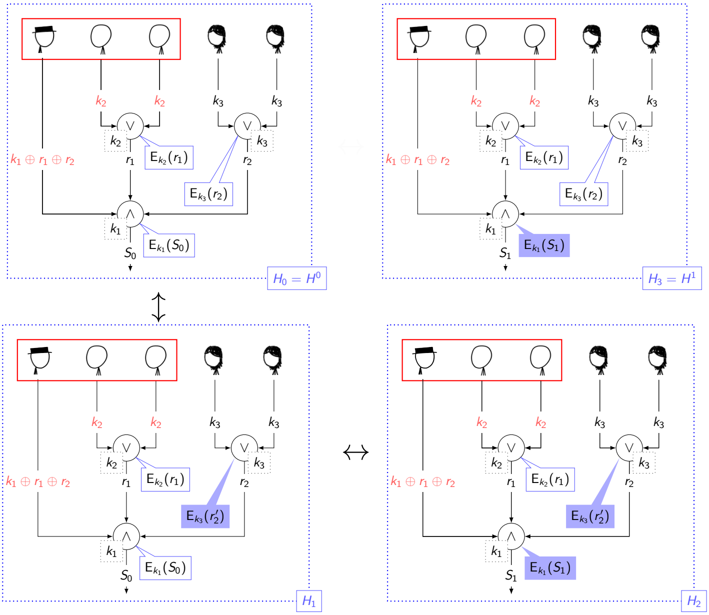

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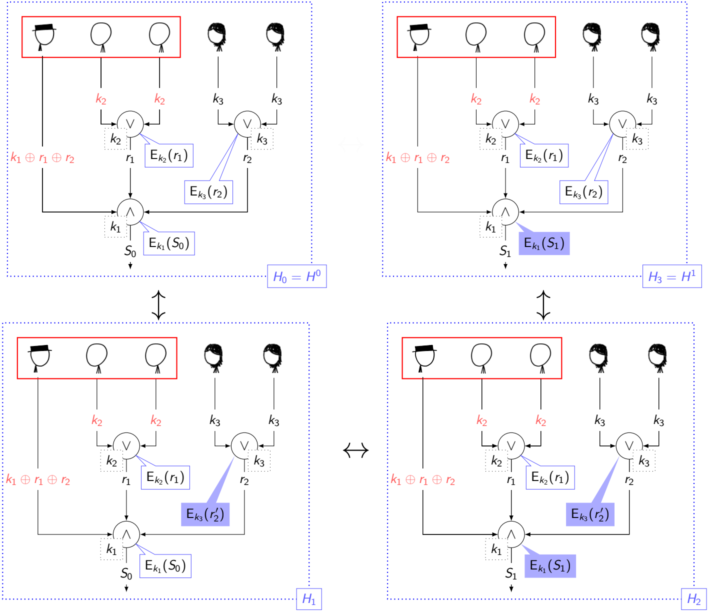

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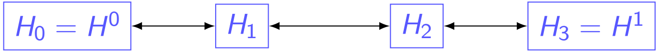

- Replace ciphertexts that the corrupt participants cannot decrypt with a bogus one
- Results in a sequence of hybrid games: the extreme games coincide with the original security game
- Show that consecutive hybrids are ϵ-indistinguishable assuming encryption is ϵ-secure: $H_i \leftrightarrow H_{i+1}$

Hybrids and Pebbling

Hybrids can be modelled using a pebbling game on the circuit

- Pebble $\implies$ bogus ciphertext/no pebble $\implies$ real ciphertext

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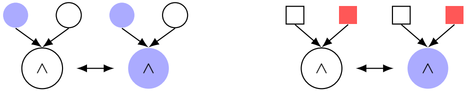

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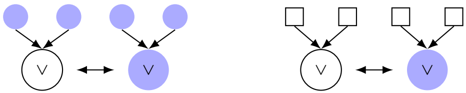
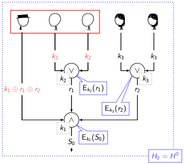
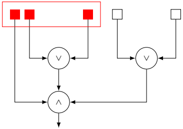


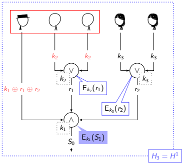
  2. gate=$\wedge$: i) one of the parents is pebbled; or ii) one of the input nodes is not corrupted

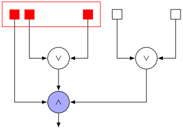
- Goal: Pebble the sink gate starting from an unpebbled state
- Pebbling sequence: $P_0, \ldots, P_\ell$, $P_i \subseteq [s]$

Hybrids and Pebbling...









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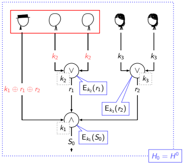
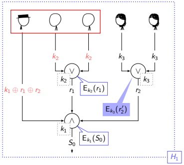
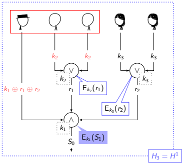
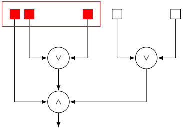
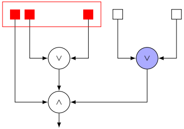
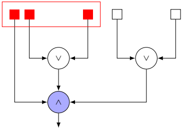




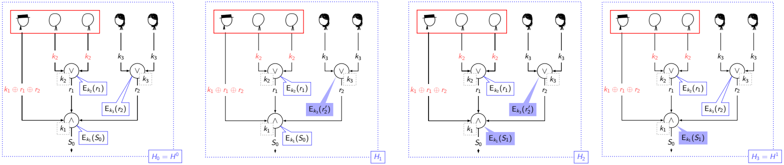


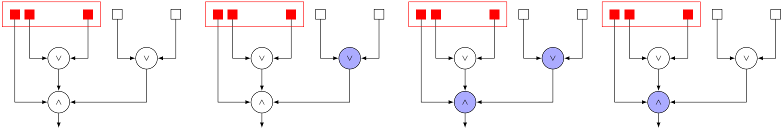






Hybrids and Pebbling...





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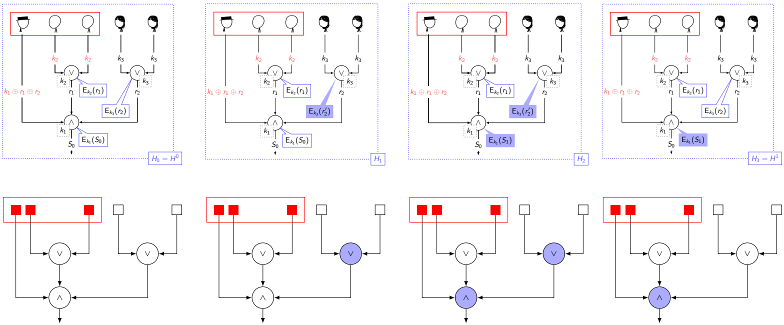

- Any valid pebbling sequence implies a sequence of hybrids!
  - $P_0, \ldots, P_\ell \Leftrightarrow H^0 = H_0, \ldots, H_\ell = H^1$
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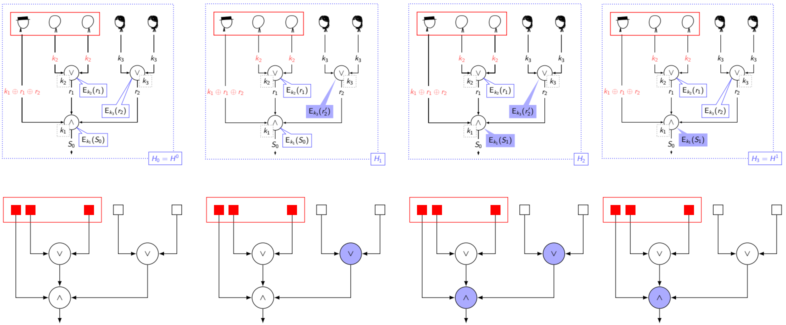

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- Corollary: if encryption scheme is ϵ-secure then Yao's scheme is $\epsilon \cdot \ell$-selectively-secure

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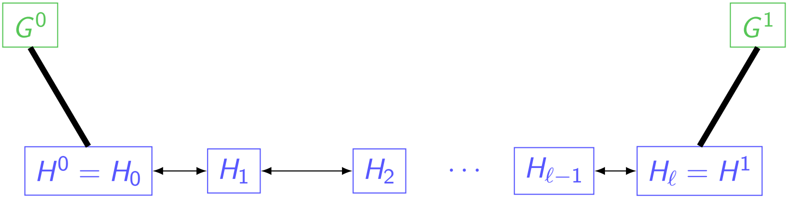
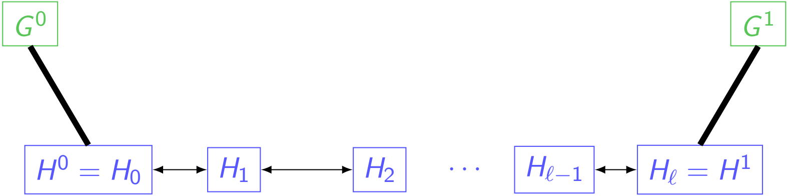
- Note: *must* know the corrupt participants

Recap



- Theorem 2 (\$): Yao's scheme is $\epsilon \cdot s$-selective-secure
- Lemma 1 (\$\$\$): ϵ-selective-secure $\implies$ $\epsilon \cdot 2^n$-adaptive-secure
- Corollary 2 (\$\$\$): Yao's scheme is $\epsilon \cdot s \cdot 2^n$-adaptive-secure

Adaptive Security: Avoiding Exponential Loss



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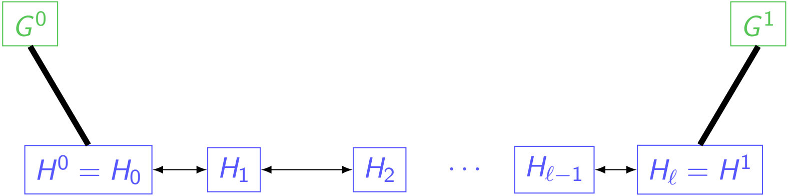

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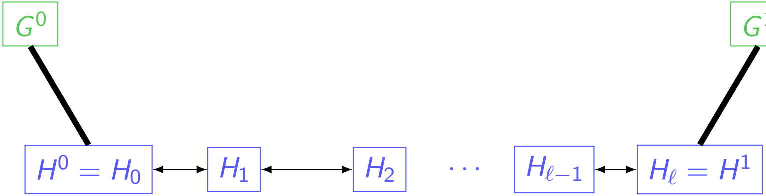

- Observation: *Can play a hybrid game if the pebbled gates in the corresponding configuration are known*
- The level-by-level pebbling requires uses too many pebbles!
- Devise a new sequence of hybrids/pebbling sequence
  - A pebbling strategy with fewer pebbles requires less information (and hence less guessing)

Adaptive Security: Avoiding Exponential Loss...

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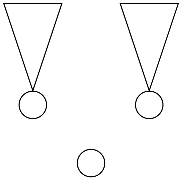

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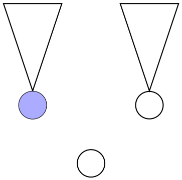

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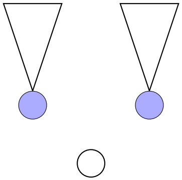

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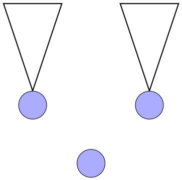

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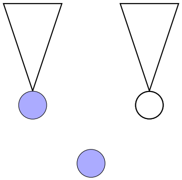

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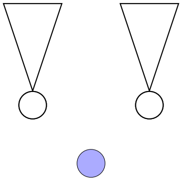

- $\#\text{moves}(d) = \#\text{moves}(d-1) \cdot 2\delta$
- $\#\text{pebbles}(d) = \#\text{pebbles}(d-1) + \delta$

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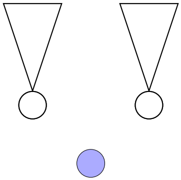

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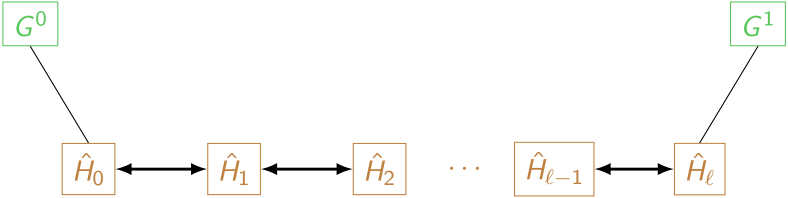
- Denoted by $\hat{P}_0, \ldots, \hat{P}_\ell$

Adaptive Security: Avoiding Exponential Loss...

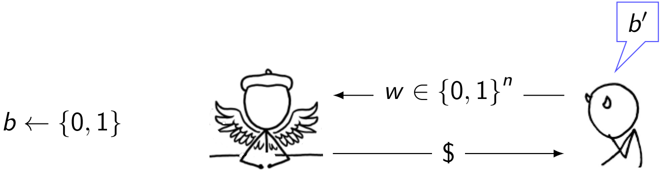


- $\hat{P}_0, \ldots, \hat{P}_\ell$ yields partially-selective hybrids $\hat{H}_0, \ldots, \hat{H}_\ell$
  - Adversary committed to a pebbling configuration instead of corrupt participants: apply random guessing
  - A pebbling configuration $\hat{P}_i$ has at most $\delta \cdot d$: probability of guessing is $2^{-(\delta \cdot d) \cdot \log s} = s^{-\delta \cdot d}$

- Theorem 1 ($$): If the encryption is ϵ-secure, then for any access structure described by a Boolean circuit of size s, depth d and fan-in/fan-out δ Yao's scheme is $\approx \epsilon \cdot (2\delta)^d \cdot s^{\delta \cdot d}$ adaptively-secure

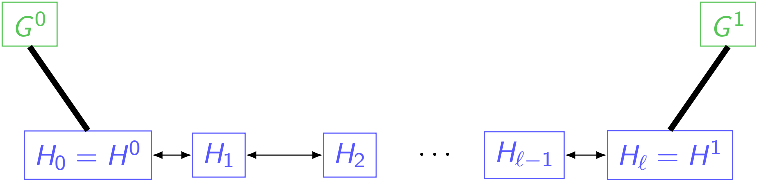
The Framework

In General



- Consider selective games where adversary commits to some information w
- Challenger checks if w consistent with observed w

In General...



- Theorem 3 (main): If the sequence of selective hybrid games $H^0 = H_0, H_1, \cdots, H_\ell = H^1$ (with $H_i \leftrightarrow H_{i+1}$) satisfy the condition that $H_i \leftrightarrow H_{i+1}$ uses only $w_i = h_i(w) \in \{0,1\}^m$ then ϵ-selective security implies $\epsilon \cdot \ell \cdot 2^m$-adaptive security

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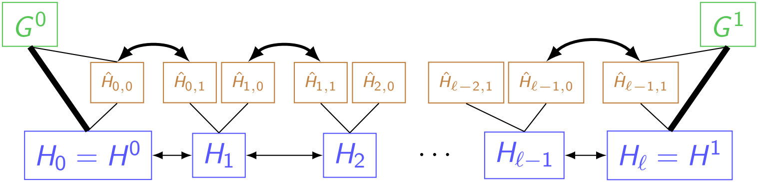

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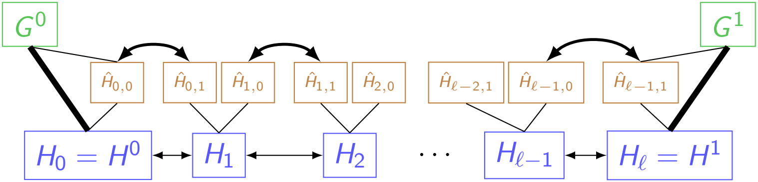

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- Results captured
  - Generalized selective decryption [P,FJP]
  - Constrained pseudo-random functions [FKPR]
  - Yao's garbled circuits [JW]

Open Questions

- Derive lower bounds from pebbling lower bounds
- Find more proofs that fit the framework

References

[BL] Benaloh and Lichter. *Generalized secret sharing and monotone functions*. Crypto'88

[FKPR] Fuchsbauer et al.. *Adaptive security of constrained PRFs*. Asiacrypt'14

[FJP] Fuchsbauer et al.. *A quasipolynomial reduction for generalized selective decryption on trees*. Crypto'15

[JKK+] Jafargohli et al.. *Be Adaptive, Avoid Overcommitting*. Crypto'17

[JW] Jafargholi and Wichs. Adaptive security of Yao's garbled circuits. TCC'16

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Thank you!